# CK0031: Homework 03

**Exercise 01.01.** Approximate the global minimum $\mathbf{x}_{☺}$ of function $f(\mathbf{x}) = x_1^4 + x_2^4 + x_1^3 + 3x_1x_2^2 - 3x_1^2 - 2x_2^2 + 10$ using the Nelder and Mead method. The contours of $f(\mathbf{x})$ are depicted below in the figure, together with the five (5) points $\mathbf{x}_{☹}^{(0)}$ that you have to use to initialise the method.

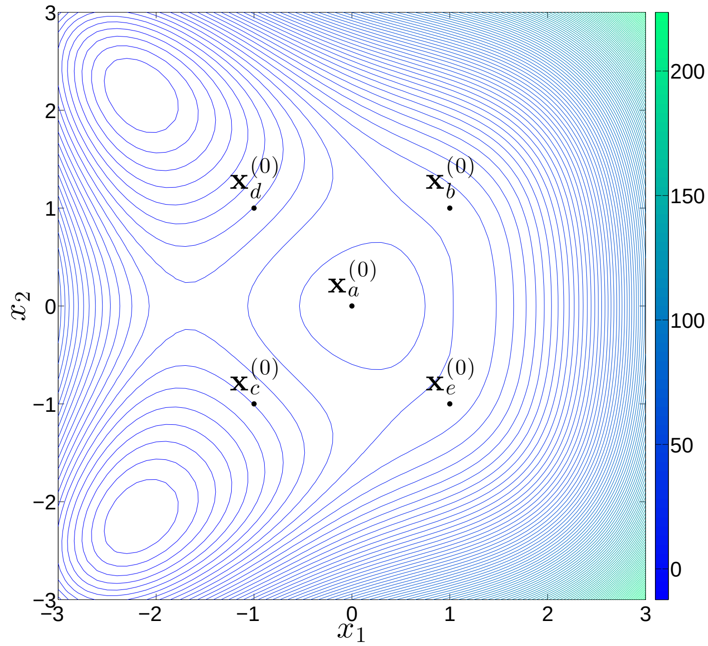


Figure 1: Function $f(\mathbf{x}) = x_1^4 + x_2^4 + x_1^3 + 3x_1x_2^2 - 3x_1^2 - 2x_2^2 + 10$ and various initial points $\mathbf{x}_{☹}^{(0)}$.

To initialise the method use the following starting points:

a) $\mathbf{x}_a^{(0)} = (0, 0)$;

b) $\mathbf{x}_{\mathbf{b}}^{(\mathbf{0})} = (+1, +1)$;

c) $\mathbf{x}_c^{(0)} = (-1, -1)$;

d) $\mathbf{x}_d^{(0)} = (-1, +1)$;

e) $\mathbf{x}_e^{(0)} = (+1, -1)$.

You are requested to briefly report and diagram on the progress of the iterations (convergence history, tolerance, etc.) and provide a main function that calls appropriate routines that implement the method (e.g., the M-function `fminsearch`, the Python function `minimize`, etc.) If you are utilising your own implementation of the method, then the relevant code must be provided, too.

[*About code*]: If the code is short (i.e., at most 3-page long), it is okay to paste it to your solution sheet. Otherwise, it is more appropriate to either package it together with your solution sheet, or provide a link in your submission for us to download it (Note: If you opt for the link, it is your responsibility to make sure that the link is functioning also after deadline.)

**Exercise 01.02.** Minimise function $f(\mathbf{x}) = (x_1^2 - x_1^3 x_2 - 2x_2 + 2x_1 x_2^2) + (3 - x_1 x_2)^2$ using the descent method with different choices of the descent direction (i.e.: i) Newton's directions; ii) quasi-Newton's directions; iii) gradient directions; and, iv) conjugate-gradient directions).

Set a tolerance $\varepsilon = 10^{-8}$ for the stopping criterion

$$\max_{1 \leq i \leq n} \left| \frac{\left[\nabla f(\mathbf{x}^{(k+1)})\right]_i \max\left(|\mathbf{x}_i^{(k+1)}|, 1\right)}{\max\left(|f(\mathbf{x}^{(k+1)})|, 1\right)} \right| \leq \varepsilon \quad (\text{with } n = 2),$$

and initial solutions $\mathbf{x}_a^{(0)} = (-1, +1)$, $\mathbf{x}_b^{(0)} = (+2, +1)$ and $\mathbf{x}_c^{(0)} = (+2, -1)$.

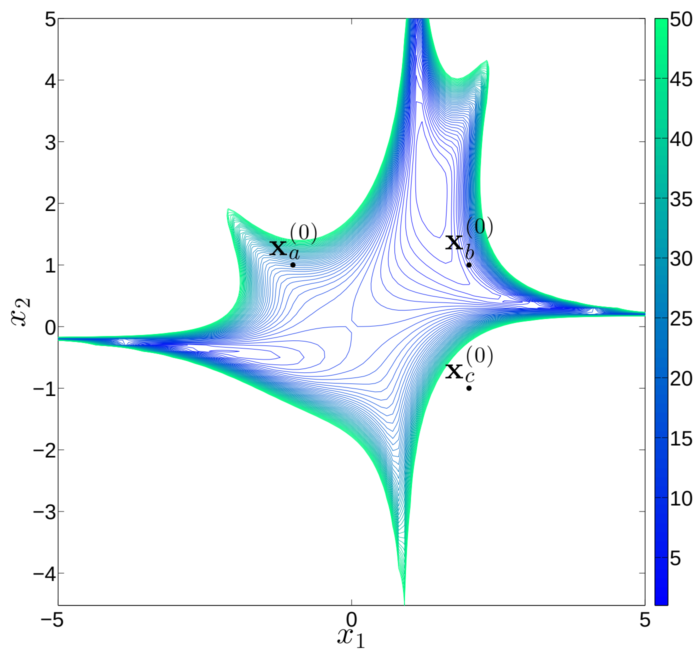


Figure 2: Function $f(\mathbf{x}) = (x_1^2 - x_1^3 x_2 - 2x_2 + 2x_1 x_2^2) + (3 - x_1 x_2)^2$ and various initial points $\mathbf{x}_{\frown}^{(0)}$.

You are requested to briefly report and diagram on the progress of the iterations (convergence history, tolerance, etc.), compare the efficiency of the methods in terms of number of performed iterations, and provide a main function that calls appropriate routines that implement the method (e.g., the M-functions included in the slides: Remember, they may contain bugs!) If you are utilising your own implementations of the methods, then the relevant code must be provided, too.

[*About code*]: Again, if the code is short (i.e., at most 3-page long), it is okay to paste it to your solution sheet. Otherwise, it is more appropriate to either package it together with your solution sheet, or provide a link in your submission for us to download it (Note: If you opt for the link, it is your responsibility to make sure that the link is functioning also after deadline.)

**Exercise 01.03.** A point moves along an elliptical trajectory $\frac{x^2}{4} + y^2 = 1$ with velocity $v(x, y) = (2x+3y+4)\sin(\pi xy + 1)$. Compute an approximation of the maximum speed reached by the point and its corresponding position, using the augmented Lagrangian method.

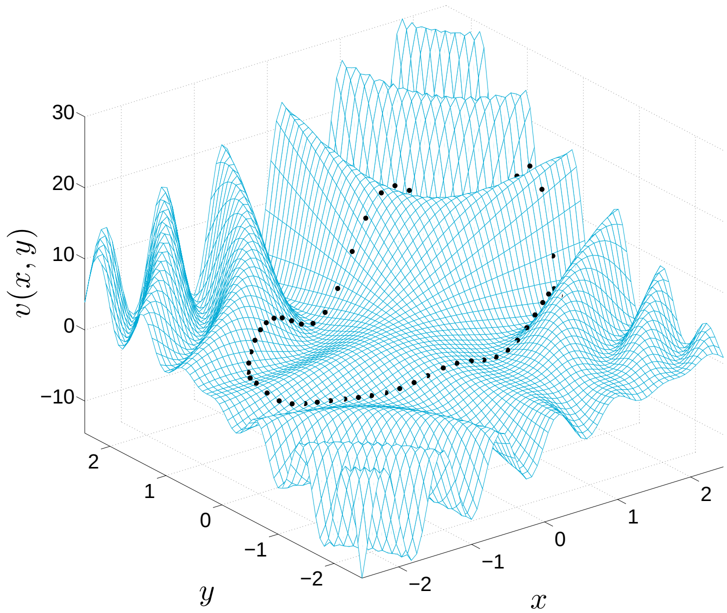


Figure 3: Function $v(x, y)$ and its restriction to the constraint $h(x, y) = \frac{x^2}{4} + y^2 - 1 = 0$.

You are requested to briefly report and diagram on the progress of the iterations (convergence history, tolerance, etc.) and provide a main function that calls appropriate routines that implement the method (e.g., the M-functions included in the slides: Remember, they may contain bugs!) If you are utilising your own implementations of the methods, relevant code must be provided, too.

[*About code*]: Again, if the code is short (i.e., at most 3-page long), it is okay to paste it to your solution sheet. Otherwise, it is more appropriate to either package it together with your solution sheet, or provide a link in your submission for us to download it (Note: If you opt for the link, it is your responsibility to make sure that the link is functioning also after deadline.)