CK0031: Homework 05

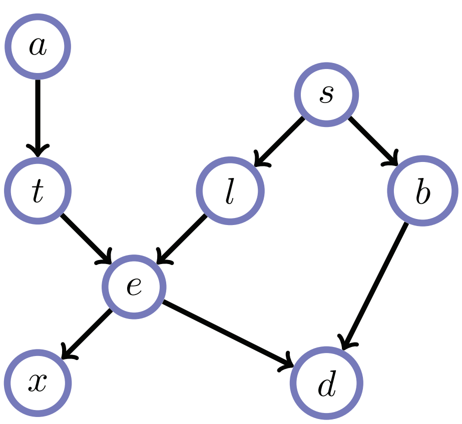
Exercise 05.00 (PRML 8.3 and 8.4). Consider three binary variables $a, b, c \in \{0, 1\}$ having the joint distribution as follows

a	b	c	$p(a,b,c)$
0	0	0	0.192
0	0	1	0.144
0	1	0	0.048
0	1	1	0.216
1	0	0	0.192
1	0	1	0.064
1	1	0	0.048
1	1	1	0.096

A: Show by direct evaluation that this distribution has the property that a and b are marginally dependent, so that $p(a,b) \neq p(a)p(b)$, but that they become independent when conditioned on c, so that $p(a,b|c) = p(a|c)p(b|c)$ for both $c = 0$ and $c = 1$.

B: Evaluate the distributions $p(a)$, $p(b|c)$, and $p(c|a)$ corresponding to the joint above and show by direct evaluation that $p(a,b,c) = p(a)p(c|a)p(b|c)$ and show the corresponding directed graph.

Exercise 05.01 (BRML 3.4). The Chest Clinic network concerns the diagnosis of lung disease (tuberculosis, lung cancer, or both, or neither), see Figure 1 and S. L. Lauritzen and D. J. Spiegelhalter: 'Local Computations with Probabilities on Graphical Structures and Their Application to Expert Systems', *Journal of the Royal Statistical Society, Series B (Methodological)*, **50**(2), 157–224, 1988. In this model a visit to Asia is assumed to increase the probability of tuberculosis.



x = Positive X-ray

d = Dyspnea (Shortness of breath)

e = Either Tuberculosis or Lung Cancer

t = Tuberculosis

l = Lung Cancer

b = Bronchitis

a = Visited Asia

s = Smoker

Figure 1: Chest clinic: Belief network structure.

A: State if these conditional independence relations are true or false and motivate your answers

A1 *Tuberculosis* $\perp\!\!\!\perp$ *Smoking*|*Shortness of breath*

A2 *Lung cancer* $\perp\!\!\!\perp$ *Bronchitis*|*Smoking*

A3 *Visit to Asia* $\perp\!\!\!\perp$ *Smoking*|*Lung cancer*

A4 *Visit to Asia* $\perp\!\!\!\perp$ *Smoking*|*Lung cancer, Shortness of breath*

B: Calculate by hand (that is, show your working) the values for $p(d)$, $p(d|s = \texttt{true})$ and $p(d|s = \texttt{false})$, the table values are

$$
\begin{aligned}
p(a = \texttt{tr}) &= 0.01 & p(s = \texttt{tr}) &= 0.50 \\
p(t = \texttt{tr}|a = \texttt{tr}) &= 0.05 & p(t = \texttt{tr}|a = \texttt{fa}) &= 0.01 \\
p(l = \texttt{tr}|s = \texttt{tr}) &= 0.10 & p(l = \texttt{tr}|s = \texttt{fa}) &= 0.01 \\
p(b = \texttt{tr}|s = \texttt{tr}) &= 0.60 & p(b = \texttt{tr}|s = \texttt{fa}) &= 0.30 \\
p(x = \texttt{tr}|e = \texttt{tr}) &= 0.98 & p(x = \texttt{tr}|e = \texttt{fa}) &= 0.05 \\
p(d = \texttt{tr}|e = \texttt{tr}, b = \texttt{tr}) &= 0.90 & p(d = \texttt{tr}|e = \texttt{tr}, b = \texttt{fa}) &= 0.70 \\
p(d = \texttt{tr}|e = \texttt{fa}, b = \texttt{tr}) &= 0.80 & p(d = \texttt{tr}|e = \texttt{fa}, b = \texttt{fa}) &= 0.10
\end{aligned}
$$

$p(e = \texttt{tr}|t, l) = 0$ only if both $t = \texttt{fa}$ and $l = \texttt{fa}$, 1 otherwise.

Exercise 05.02 (BRML 3.12). You are given two belief networks represented as DAGs $\mathcal{A}$ and $\mathcal{B}$ with associated adjacency matrices $\mathbf{A}$ and $\mathbf{B}$. Write your own code that takes the two matrices $\mathbf{A}$ and $\mathbf{B}$ as inputs and outputs `1` if $\mathcal{A}$ and $\mathcal{B}$ are Markov equivalent, and `0` otherwise.

Exercise 05.03 (BRML 3.13). You are given the adjacency matrices of two belief networks:

$$
\mathbf{A} = \begin{pmatrix}
0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\text{ and } \mathbf{B} = \begin{pmatrix}
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}. \tag{1}
$$

Use the code you have written for Exercise 05.02 to state if they are Markov equivalent.

[*About code*]: As always, if the code is short (i.e., at most 3-page long), it is okay to paste it to your solution sheet. Otherwise, it is more appropriate to either package it together with your solution sheet, or provide a link in your submission for us to download it (Note: If you opt for the link, it is your responsibility to make sure that the link is functioning also after deadline.)

[*About the BRMLtoolbox*]: An official and full Matlab implementation of the toolbox exists. It comes in two flavours: i) object-oriented and ii) non object-oriented. The non-OO implementation is expected to work also with GNU Octave. The official Julia implementation is still incomplete. There is also, at least, one non-official and still incomplete Python implementation.

- `Official Matlab OO, Matlab non-OO and Julia implementations`;
- `Unofficial Python implementation`.