Matrícula (First-name Last-name)

CK0031/CK0248: Homework C

Exercise A.1. Suppose that your are given the objective function $f(\mathbf{x}) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$.

- Sketch the contour lines of $f(\mathbf{x})$ and visually identify the local minimiser $\mathbf{x}^*$
- Compute the gradient $\boldsymbol{\nabla} f(\mathbf{x})$ and the Hesse matrix $\boldsymbol{\nabla}^2 f(\mathbf{x})$ at $\mathbf{x} = (0, 0)^T$
- Compute the gradient $\boldsymbol{\nabla} f(\mathbf{x})$ and the Hesse matrix $\boldsymbol{\nabla}^2 f(\mathbf{x})$ at $\mathbf{x} = (1, 1)^T$

Show all the derivations and the resulting expressions.

Exercise A.2. Suppose that you are given the objective function $f(\mathbf{x}) = 10(x_2 - x_1^2)^2 + (1 - x_1)^2$.

- Sketch the contour lines of $f(\mathbf{x})$ and visually identify the local minimiser $\mathbf{x}^*$
- Compute expressions for the gradient $\boldsymbol{\nabla} f(\mathbf{x})$ and the Hesse matrix $\boldsymbol{\nabla}^2 f(\mathbf{x})$

Consider the direction $\mathbf{d} = -\boldsymbol{\nabla} f(\mathbf{x})$

- Show that $\mathbf{d}$ is a descent direction at point $\mathbf{x} = (0, 1)^T$
- Calculate the position of point $\mathbf{x}_{\text{new}} = \mathbf{x} + \alpha \mathbf{d}$, for $\alpha = 0.1$
- Calculate the position of point $\mathbf{x}_{\text{newest}} = \mathbf{x}_{\text{new}} + \alpha \mathbf{d}$, for the same α

Consider the direction $\mathbf{d} = -[\boldsymbol{\nabla}^2 f(\mathbf{x})]^{-1} \boldsymbol{\nabla} f(\mathbf{x})$

- Show that $\mathbf{d}$ is a descent direction at point $\mathbf{x} = (0, 1)^T$
- Calculate the position of point $\mathbf{x}_{\text{new}} = \mathbf{x} + \alpha \mathbf{d}$, for $\alpha = 0.1$
- Calculate the position of point $\mathbf{x}_{\text{newest}} = \mathbf{x}_{\text{new}} + \alpha \mathbf{d}$, for the same α

Show all the derivations and the resulting expressions[1].

[1] A simple formula for calculating the inverse of a 2×2 matrix A

$$\text{Let } A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \text{ then } A^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$