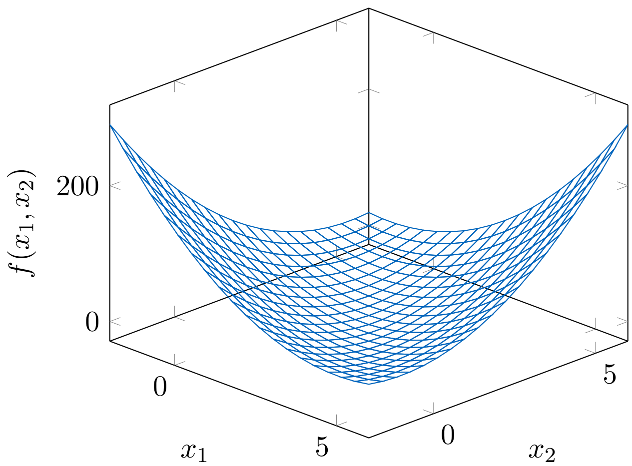
CK0031/CK0248: AP-02-II (24 de novembro de 2017)

Questão 01. You are given the objective function $f(x_1, x_2) = (x_1 + 2x_2 - 7)^2 + (2x_1 + x_2 - 5)^2$ (see figure)[1]. You are requested to find its minimiser $\mathbf{x}^*$ using a descent-direction method.



Let $\mathbf{x}^{(0)} = (0, 0)'$ be the initial solution and let $\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \alpha \mathbf{d}$ $(k = 0, 1, \dots)$ be the general structure of line-search methods, with $\mathbf{d}$ the search direction and $\alpha = 1$ the fixed step-length.

- $\rightsquigarrow$ (20%) Calculate expressions for the gradient vector $\nabla f(\mathbf{x})$ and the Hessian matrix $\nabla^2 f(\mathbf{x})$
- $\rightsquigarrow$ (40%) Calculate the first 3 iterates ($\mathbf{x}^{(k)}$ and $f\big[\mathbf{x}^{(k)}\big]$, $k = 1, 2, 3$) using the gradient method

$$\mathbf{d} = -\nabla f\big[\mathbf{x}^{(k)}\big]$$

- $\rightsquigarrow$ (40%) Calculate the first 3 iterates ($\mathbf{x}^{(k)}$ and $f\big[\mathbf{x}^{(k)}\big]$, $k = 1, 2, 3$) using the Newton method[2]

$$\mathbf{d} = -\Big[\nabla^2 f\big[\mathbf{x}^{(k)}\big]\Big]^{-1} \nabla f\big[\mathbf{x}^{(k)}\big]$$

[1] $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$.

[2] For calculating the inverse of a 2×2 matrix A

$$\text{Let } A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \text{ then } A^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$