Matrícula (First-name Last-name) **CK0031/CK0248: Homework 01**

Exercise 1. Consider the following four four-dimensional vectors $\{\mathbf{x}_i\}_{i=1}^4$ (with $\mathbf{x}_i \in \mathcal{R}^4$)

$$\mathbf{x}_1 = \begin{bmatrix} +0.5377 \\ +0.3188 \\ +3.5784 \\ +0.7254 \end{bmatrix}; \quad \mathbf{x}_2 = \begin{bmatrix} -2.2588 \\ -0.4336 \\ -1.3499 \\ +0.7147 \end{bmatrix}; \quad \mathbf{x}_3 = \begin{bmatrix} -2.2588 \\ -0.4336 \\ -1.3499 \\ +0.7147 \end{bmatrix}; \quad \mathbf{x}_4 \begin{bmatrix} +0.8622 \\ +0.3426 \\ +3.0349 \\ -0.2050 \end{bmatrix}.$$

For all $i, j \in \{1, 2, 3, 4\}$, write code to compute and store

1. The vector $\mathbf{l}_1$ of all L_1 vector norms, $[\mathbf{l}_1]_i = \|\mathbf{x}_i\|_1$
2. The vector $\mathbf{l}_2$ of all L_2 vector norms, $[\mathbf{l}_2]_i = \|\mathbf{x}_i\|_2$
3. The matrices $\mathbf{\Delta}_i$ of all pairwise differences, $[\mathbf{\Delta}_i]_j = \mathbf{x}_i - \mathbf{x}_j$
4. The matrices $\mathbf{\Sigma}_i$ of all pairwise summations, $[\mathbf{\Sigma}_i]_j = \mathbf{x}_i + \mathbf{x}_j$
5. The matrix $\mathbf{K}$ of all pairwise inner products, $[\mathbf{K}]_{ij} = \mathbf{x}_i \cdot \mathbf{x}_j$
6. The matrix $\mathbf{D}$ of all pairwise vector distances, $[\mathbf{D}]_{ij} = \|\mathbf{x}_i - \mathbf{x}_j\|_2$

Let $\mathbf{x}_i^{(k)} = \mathbf{x}_i$ for $k = 1$ and for all $i \in \{1, 2, 3, 4\}$. Write code to compute and store

1. $\mathbf{x}_i^{(k+1)} = \mathbf{x}_i^{(k)} + \dfrac{1}{k}\mathbf{x}_i^{(k)}$, for $k = 1, 2, \cdots \rightsquigarrow$
2. $\mathbf{x}_i^{(k+1)} = \mathbf{x}_i^{(k)} - \dfrac{1}{k}\mathbf{x}_i^{(k)}$, for $k = 1, 2, \cdots \rightsquigarrow$

Iterate until k is such that $\|\mathbf{x}_i^{(k)} - \mathbf{x}_i^{(k-1)}\|_2 \leq \varepsilon$ or $k = 10K$, for some choice of ε.