

**Exercise 1.** Consider the following matrices

$$\mathbf{A} = \begin{bmatrix} +0.537 & +0.862 & -0.433 \\ +1.833 & +0.318 & +0.342 \\ -2.258 & -1.307 & +3.578 \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} -0.063 & -0.124 \\ +0.714 & +1.489 \\ -0.205 & +1.409 \end{bmatrix}; \quad \mathbf{C} = \begin{bmatrix} +1.417 & -1.207 & +1.630 \\ +0.671 & +0.717 & +0.488 \end{bmatrix};$$

$$\mathbf{a} = \begin{bmatrix} +0.862 \\ +0.342 \\ +3.034 \\ -0.205 \end{bmatrix}; \quad \mathbf{b} = [+0.862 \quad +0.346 \quad +3.039]; \quad \mathbf{c} = \begin{bmatrix} +0.822 \\ +0.326 \\ +3.039 \\ -0.250 \end{bmatrix}.$$

Write code to compute and store some of the following operations:

1. The matrices  $\mathbf{A} + 2\mathbf{A} - (1/3)\mathbf{A}$ ,  $\mathbf{B} - 2\mathbf{B} + (1/3)\mathbf{B}$ ,  $\mathbf{C} + 2\mathbf{C} + 3\mathbf{C}$ .
2. The matrices  $\mathbf{aa}$ ,  $\mathbf{ab}$ ,  $\mathbf{ac}$ ;  $\mathbf{ba}$ ,  $\mathbf{bb}$ ,  $\mathbf{bc}$ ;  $\mathbf{ca}$ ,  $\mathbf{cb}$ ,  $\mathbf{cc}$ .
3. The matrices  $\mathbf{aa}^T$ ,  $\mathbf{ab}^T$ ,  $\mathbf{ac}^T$ ;  $\mathbf{ba}^T$ ,  $\mathbf{bb}^T$ ,  $\mathbf{bc}^T$ ;  $\mathbf{ca}^T$ ,  $\mathbf{cb}^T$ ,  $\mathbf{cc}^T$ .
4. The matrices  $\mathbf{a}^T\mathbf{a}$ ,  $\mathbf{a}^T\mathbf{b}$ ,  $\mathbf{a}^T\mathbf{c}$ ;  $\mathbf{b}^T\mathbf{a}$ ,  $\mathbf{b}^T\mathbf{b}$ ,  $\mathbf{b}^T\mathbf{c}$ ;  $\mathbf{c}^T\mathbf{a}$ ,  $\mathbf{c}^T\mathbf{b}$ ,  $\mathbf{c}^T\mathbf{c}$ .
5. The matrices  $\mathbf{AA}$ ,  $\mathbf{AB}$ ,  $\mathbf{AC}$ ;  $\mathbf{BA}$ ,  $\mathbf{BB}$ ,  $\mathbf{BC}$ ;  $\mathbf{CA}$ ,  $\mathbf{CB}$ ,  $\mathbf{CC}$ .
6. The matrices  $\mathbf{AA}^T$ ,  $\mathbf{AB}^T$ ,  $\mathbf{AC}^T$ ;  $\mathbf{BA}^T$ ,  $\mathbf{BB}^T$ ,  $\mathbf{BC}^T$ ;  $\mathbf{CA}^T$ ,  $\mathbf{CB}^T$ ,  $\mathbf{CC}^T$ .
7. The matrices  $\mathbf{A}^T\mathbf{A}$ ,  $\mathbf{A}^T\mathbf{B}$ ,  $\mathbf{A}^T\mathbf{C}$ ;  $\mathbf{B}^T\mathbf{A}$ ,  $\mathbf{B}^T\mathbf{B}$ ,  $\mathbf{B}^T\mathbf{C}$ ;  $\mathbf{C}^T\mathbf{A}$ ,  $\mathbf{C}^T\mathbf{B}$ ,  $\mathbf{C}^T\mathbf{C}$ .
8. The matrices  $\mathbf{aA}$ ,  $\mathbf{aB}$ ,  $\mathbf{aC}$ ;  $\mathbf{bA}$ ,  $\mathbf{bB}$ ,  $\mathbf{bC}$ ;  $\mathbf{cA}$ ,  $\mathbf{cB}$ ,  $\mathbf{cC}$ .
9. The matrices  $\mathbf{a}^T\mathbf{A}$ ,  $\mathbf{a}^T\mathbf{B}$ ,  $\mathbf{a}^T\mathbf{C}$ ;  $\mathbf{b}^T\mathbf{A}$ ,  $\mathbf{b}^T\mathbf{B}$ ,  $\mathbf{b}^T\mathbf{C}$ ;  $\mathbf{c}^T\mathbf{A}$ ,  $\mathbf{c}^T\mathbf{B}$ ,  $\mathbf{c}^T\mathbf{C}$ .
10. The matrices  $\mathbf{aA}^T$ ,  $\mathbf{aB}^T$ ,  $\mathbf{aC}^T$ ;  $\mathbf{bA}^T$ ,  $\mathbf{bB}^T$ ,  $\mathbf{bC}^T$ ;  $\mathbf{cA}^T$ ,  $\mathbf{cB}^T$ ,  $\mathbf{cC}^T$ .
11. The matrices  $\mathbf{Aa}$ ,  $\mathbf{Ab}$ ,  $\mathbf{Ac}$ ;  $\mathbf{Ba}$ ,  $\mathbf{Bb}$ ,  $\mathbf{Bc}$ ;  $\mathbf{Ca}$ ,  $\mathbf{Cb}$ ,  $\mathbf{Cc}$ .
12. The matrices  $\mathbf{A}^T\mathbf{a}$ ,  $\mathbf{A}^T\mathbf{b}$ ,  $\mathbf{A}^T\mathbf{c}$ ;  $\mathbf{B}^T\mathbf{a}$ ,  $\mathbf{B}^T\mathbf{b}$ ,  $\mathbf{B}^T\mathbf{c}$ ;  $\mathbf{C}^T\mathbf{a}$ ,  $\mathbf{C}^T\mathbf{b}$ ,  $\mathbf{C}^T\mathbf{c}$ .
13. The matrices  $\mathbf{Aa}^T$ ,  $\mathbf{Ab}^T$ ,  $\mathbf{Ac}^T$ ;  $\mathbf{Ba}^T$ ,  $\mathbf{Bb}^T$ ,  $\mathbf{Bc}^T$ ;  $\mathbf{Ca}^T$ ,  $\mathbf{Cb}^T$ ,  $\mathbf{Cc}^T$ .

Assuming that (NumPy) array broadcasting is not permitted, indicate unfeasible operations.