

Exercise A.1. Consider the sum-of-squares error function

$$E(\mathbf{w}) = \frac{1}{2} \sum_{n=1}^N \left(y(x_n, \mathbf{w}) - t_n \right)^2, \quad (1)$$

in which the function $y(x, \mathbf{w})$ is given by the polynomial

$$y(x, \mathbf{w}) = w_0 + w_1x + w_2x^2 + \cdots + w_Mx^M = \sum_{j=0}^M w_jx^j. \quad (2)$$

Show that the coefficients $\mathbf{w} = \{w_i\}$ that minimise this error function are given by the solution of the following set of linear equations

$$\sum_{j=0}^M A_{ij}w_j = T_i, \quad (3)$$

where

$$A_{ij} = \sum_{n=1}^N (x_n)^{j+1}; \quad (4a)$$

$$T_i = \sum_{n=1}^N (x_n)^i t_n. \quad (4b)$$

Suffix i or j denotes the index of a component, whereas $(x)^i$ denotes x raised to the power of i .

Solution: Substituting Equation 2 into Equation 1 and then differentiating w.r.t. w_j , we obtain

$$\sum_{n=1}^N \left(\sum_{j=0}^M w_j x_n^j - t_n \right) x_n^i = 0.$$

Rearranging terms gives the required result.

Exercise A.2. ...

Solution: ...