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CK0255: Homework A

Exercise A.1. You are given a random sample $X_1, X_2, \ldots, X_n$ from a population with PDF $f(x|\theta)$. Show that the maximising the likelihood function $\mathcal{L}(\theta|\mathbf{x})$, as a function of θ is equivalent to maximising $\log[\mathcal{L}(\theta|\mathbf{x})]$.

Solution: The log function is a strictly monotone increasing function. Therefore, $\mathcal{L}(\theta|\mathbf{x}) > \mathcal{L}(\theta'|\mathbf{x})$ if and only if $\log[\mathcal{L}(\theta|\mathbf{x})] > \log[\mathcal{L}(\theta'|\mathbf{x})]$.

So, the value $\hat{\theta}$ that maximises $\log[(\theta|\mathbf{x})]$ is the same as the value that maximises $\mathcal{L}(\theta|\mathbf{x})$.

Exercise A.2. Write code to generate 1000 variables from the following distributions

1. $Y \sim \text{Binomial}(8, 2/3)$;
2. $Y \sim \text{Hyper-geometric}(\mathrm{N} = 10, \mathrm{M} = 8, \mathrm{K} = 4)$;

Compare the mean and the variance with the theoretical values.

Solution: The `R` code and corresponding output is the following

1. $\rightsquigarrow$

```r
obs <- rbinom(1000,8,2/3)
meanobs <- mean(obs)
variance <- var(obs)
```

```
Output:
 > meanobs
   [1] 5.231

 > variance
   [1] 1.707346
```

2. $\rightsquigarrow$

```r
obs<- rhyper(1000,8,2,4)
meanobs <- mean(obs)
variance <- var(obs)
```

```
Output:
 > meanobs
   [1] 3.169

 > variance
   [1] 0.4488879
```