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CK0255/TIP8421: Homework A

Exercise A.1. Find the union and the intersection of two sets C_1 and C_2 in which

- $C_1 = \{(x, y) : 0 < x < 2, 1 < y < 2\}$;
- $C_2 = \{(x, y) : 1 < x < 3, 1 < y < 3\}$.

Exercise A.2. Let $C_1, C_2, C_3, \ldots$ be a sequence of sets.

- If the sequence is such that $C_k \subset C_{k+1}, k = 1, 2, 3, \ldots,$ we call the sequence non-decreasing;
- If the sequence is such that $C_k \supset C_{k+1}, k = 1, 2, 3, \ldots,$ we call the sequence non-increasing.

Give an example of these kinds of sequences.

Exercise A.3. Let $C_1, C_2, C_3, \ldots$ be sets such that $C_k \supset C_{k+1}, k = 1, 2, 3, \ldots$.

$\lim_{k\uparrow\infty} C_k$ is defined as the intersection $C_1 \cap C_2 \cap C_3 \cap \cdots$.

Find $\lim_{k\uparrow\infty} C_k$ if $C_k = \{x : 2 < x \leq [2 + 1/k]\}, k = 1, 2, 3, \ldots$.

Exercise A.4. Consider the set of points that are inside or on the boundary of the unit square.

Let $Q(C) = \int_c\int \mathrm{d}x\mathrm{d}y$. Compute $Q(C)$ when $C \subset \mathcal{C}$ is the set $\{(x, y) : 0 < x/2 \leq y \leq 3x/2 < 1\}$.

Exercise A.5. Let $\mathcal{C} = \{c : -\infty < c < \infty\}$ be the sample space and let $C \subset \mathcal{C}$ be a set for which $\int_C f(x)\mathrm{d}x$ with $f(x) = e^{-|x|}$ exists. Show that this integral is not a valid probability set function.

By what constant we must multiply the integrand, to make it a valid probability set function?