Matrícula (First-name Last-name) **CK0255/TIP8421: Homework B**

Exercise A.1. Find the constant c such that the following expressions of $p(x)$ are valid probability mass functions for the random variable X

- $p(x) = c(2/3)^x$ for $x = 1, 2, 3, \dots$ and zero elsewhere.
- $p(x) = cx$ for $x = 1, 2, 3, 4, 5, 6$ and zero else where.

Exercise A.2. Let the random variable X have the probability mass function $p_X(x) = (1/2)^x$ for $x = 1, 2, 3, \dots$.

- Determine the PMF $p_Y(y)$ of the random variable $Y = X^3$.
- Plot $p_X(x)$ and $p_Y(y)$.

Exercise A.3. Let the random variable X have the probability density function $f_X(x) = 1/3$ for $x \in (-1, 2)$ and zero elsewhere.

- Determine the PDF $f_Y(y)$ and the CDF $F_Y(y)$ of the random variable $Y = X^2$.

Exercise A.4. Let the random variable X have the PDF $f_X(x) = 1/\pi$ for $x \in (-\pi/2, \pi/2)$.

- Determine the PDF $f_Y(y)$ of the random variable $Y = \tan(X)$ and plot its graph.

Exercise A.5. A *mode* of a distribution of a RV X is a value x that maximises the PDF/PMF.

Consider a random variable X whose CDF $F(x) = 1 - e^{-x} - xe^{-x}$ for $x \in [0, \infty)$ and zero elsewhere.

- Determine the PDF $f(x)$ and the mode of the distribution.