Matrícula (First-name Last-name) **CK0255/TIP8421: Assigment I**

Exercise 1. Soap bubbles are blown from a device and explode at a distance D, a real number with an exponential distribution with parameter β. Bubble explosions can be observed only if they occur within a frame extending from $d = 1$ [m] to $d = 20$ [m].

A number N of explosions are observed at distances $\{d_1, \dots, d_N\}$

$$\{d_n\}_{n=1}^{N} = \{1.5, 2, 3, 4, 5, 12\}.$$

Write down the probability $p(d|\beta)$ of one distance d, given β.

0.1) Plot $p(d|\beta)$ as a function of d, for some values of β.

0.2) Plot $p(d|\beta)$ as a function of β, for some values of d.

0.3) Plot $p(d|\beta)$ as a function of both d and β.

Comment on the probability density function $p(d|\beta)$ and on the plots you obtained[1].

For the given data, write down the likelihood function $p(\{d\}|\beta)$.

1.1) Plot the likelihood function of each data point d_n.

1.2) Plot the likelihood function of all data $\{d\}$.

Comment on the plots you obtained and determine the value of β that maximises the likelihood[2].

Exercise 2. You are given a random sample $\{(X_1^{(n)}, X_2^{(n)}, X_3^{(n)}, X_4^{(n)}, X_5^{(n)})' = \mathbf{X}^{(n)}\}_{n=1}^{1024}$ in which each $\mathbf{X}^{(n)} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ with unknown parameters $\boldsymbol{\mu} \in \mathcal{R}^5$ and $\boldsymbol{\Sigma} \in \mathcal{R}^{5\times 5}$. Get the data here.

Firstly, estimate the parameters of this multivariate normal using the maximum likelihood principle. Then, you use the estimates to determine the distribution of the following random variables/vectors:

2.1) X_2 and $X_2|\mathbf{X}_{1,3,4,5} = (x_1, x_3, x_4, x_5)'$

2.2) $\mathbf{X}_{1,4}$ and $\mathbf{X}_{1,4}|\mathbf{X}_{2,3,5} = (x_2, x_3, x_5)'$

You are free to choose your favourite real values of x_1, x_2, x_3, x_4 and x_5.

Plot all the obtained probability density functions and comment on the results.

[1] [*Tip*]: $1/\beta$ has the units of distance.
[2] [*Tip*]: Logarithmic scales can help.