CK0255/TIP8244: Homework A

Exercise A.1. You are given a random sample $X_1, X_2, \ldots, X_n$ from a population with PDF $f(x|\theta)$. Show that the maximising the likelihood function $\mathcal{L}(\theta|\mathbf{x})$, as a function of θ is equivalent to maximising $\log [\mathcal{L}(\theta|\mathbf{x})]$.

Solution: The log function is a strictly monotone increasing function. Therefore, $\mathcal{L}(\theta|\mathbf{x}) > \mathcal{L}(\theta'|\mathbf{x})$ if and only if $\log [\mathcal{L}(\theta|\mathbf{x})] > \log [\mathcal{L}(\theta'|\mathbf{x})]$.

So, the value $\hat{\theta}$ that maximises $\log [(\theta|\mathbf{x})]$ is the same as the value that maximises $\mathcal{L}(\theta|\mathbf{x})$.

Exercise A.2. Write code to generate 1000 variables from the following distributions

1. $Y \sim \text{Binomial}(8, 2/3)$;

2. $Y \sim \text{Hyper-geometric}(N = 10, M = 8, K = 4)$;

Compare the mean and the variance with the theoretical values.

Solution: The R code and corresponding output is the following

1. ⤳

```
1 obs <- rbinom (1000 ,8 ,2/3)
2 meanobs <- mean (obs)
3 variance <- var (obs)
```

```
1 Output :
2  > meanobs
3    [1]  5.231
4
5  > variance
6    [1]  1.707346
```

2. ⤳

```
1 obs<- rhyper (1000 ,8 ,2 ,4)
2 meanobs <- mean (obs)
3 variance <- var (obs)
```

```
1 Output :
2  > meanobs
3    [1]  3.169
4
5  > variance
6    [1]  0.4488879
```