September 25, 2018

CK0255/TIP8244: EX 01

Exercise 01. Plot the graph of the following function

$$f(t) = \delta_{-1}(t) - (t-2)\delta_{-1}(t-2) + (t-3)\delta_{-1}(t-5).$$

Exercise 02. Consider the following system given in IO representation

$$\dot{y}(t) + y(t) + \alpha = u(t), \quad \alpha \in \mathcal{R},$$

Discuss the linearity and stationarity properties of the systems as a function of the parameter α.

Exercise 03. An IO model has a characteristic polynomial with 3 distinct roots, each of which associates with a stable mode and a time constant that is smaller than 1 $[s]$. Comment on where in the complex plane these roots are located.

Exercise 04. Consider the following linear time-invariant system in IO representation

$$2\frac{\mathrm{d}^2 y(t)}{\mathrm{d}t^2} + 4\frac{\mathrm{d}y(t)}{\mathrm{d}t} + 2y(t) = u(t)$$

Determine and plot the force-free response from $t_0 = 0$ and the following initial conditions

$$\begin{aligned} y(t)\Big|_{t=t_0} &= 1 \\ \frac{\mathrm{d}y(t)}{\mathrm{d}t}\Big|_{t=t_0} &= 1 \end{aligned}.$$

Exercise 05. Consider the linear time-invariant system in IO representation

$$\frac{\mathrm{d}^3 y(t)}{\mathrm{d}t^3} + 6\frac{\mathrm{d}^2 y(t)}{\mathrm{d}t^2} + 13\frac{\mathrm{d}y(t)}{\mathrm{d}t} = 2\frac{\mathrm{d}^2 u(t)}{\mathrm{d}t^2} + 3u(t)$$

1. Determine the characteristic polynomial and its roots;
2. Determine the modes of the system, classify and plot them;
3. Let $t_0 = 0$, determine the force-free evolution from initial conditions

$$\begin{aligned} y(t)\Big|_{t=t_0} &= 1 \\ \frac{\mathrm{d}y(t)}{\mathrm{d}t}\Big|_{t=t_0} &= 1. \\ \frac{\mathrm{d}^2 y(t)}{\mathrm{d}t^2}\Big|_{t=t_0} &= 1 \end{aligned}$$

Exercise 06. Consider the linear time-invariant system in IO representation

$$2\frac{\mathrm{d}^2 y(t)}{\mathrm{d}t^2} + 4\frac{\mathrm{d}y(t)}{\mathrm{d}t} + 2y(t) = 3\frac{\mathrm{d}u(t)}{\mathrm{d}} + u(t)$$

Define and determine the system's impulse evolution.