November 07, 2018

CK0255/TIP8244: EX 02

Exercise 01. Consider a linear and stationary system in SS representation

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)\\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)\end{aligned}$$

Consider the following system matrices

$$\mathbf{A} = \begin{bmatrix}1 & 3\\ 2 & 0\end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix}2\\ 1\end{bmatrix}; \quad \mathbf{C} = \begin{bmatrix}0 & 3\end{bmatrix}; \quad \mathbf{D} = \begin{bmatrix}0\end{bmatrix}$$

1. Compute the eigenvalues and the eigenvectors of matrix $\mathbf{A}$;
2. Determine the modes of matrix $\mathbf{A}$ and plot their time evolution;
3. Use the Sylvester expansion to compute the state transition matrix;
4. Discuss the existence of a similarity transformation $\mathbf{x}(t) = \mathbf{P}\mathbf{z}(t)$ that leads to a diagonal representation of the state matrix. Show the resulting state matrix, if it exists.

Exercise 02. Consider a linear and stationary system in SS representation

$$\begin{aligned}\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} &= \begin{bmatrix}0 & 4\\ 0 & -4\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + \begin{bmatrix}0 & 1\\ 3 & -1\end{bmatrix}\begin{bmatrix}u_1(t)\\ u_2(t)\end{bmatrix}\\ y(t) &= \begin{bmatrix}2 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + \begin{bmatrix}1 & 1\end{bmatrix}\begin{bmatrix}u_1(t)\\ u_2(t)\end{bmatrix}\end{aligned}$$

1. Use the Sylvester expansion to determine the state transition matrix;
2. Determine a similarity transformation that leads to a state matrix in diagonal form. Show the resulting state matrix, if it exists.

Exercise 03. Consider the linear time-invariant system given in SS representation

$$\begin{aligned}\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} &= \begin{bmatrix}-3 & 1\\ 1 & -3\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + \begin{bmatrix}1\\ -1\end{bmatrix}u(t)\\ y(t) &= \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}\end{aligned}.$$

- Determine a new representation in which the new state variables are

$$\begin{aligned}z_1(t) &= x_1(t) + x_2(t)\\ z_2(t) &= x_1(t) - x_2(t)\end{aligned};$$

- Determine the corresponding similarity transformation $\mathbf{z} = \mathbf{P}^{-1}\mathbf{x}$ and calculate all the system matrices in the new representation;
- In the original representation, let $\mathbf{x}(0) = (4, 2)^T$. Determine the state transition matrix and the force-free evolution of the system from $\mathbf{x}(0)$.

Exercise 04. Consider the linear time-invariant system given in SS representation

$$\begin{aligned}\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} &= \begin{bmatrix}0 & 1\\ 0 & -1\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + \begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}\begin{bmatrix}u_1(t)\\ u_2(t)\end{bmatrix}\\ y(t) &= \begin{bmatrix}5 & 2\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}\begin{bmatrix}0 & 7\end{bmatrix}\begin{bmatrix}u_1(t)\\ u_2(t)\end{bmatrix}\end{aligned}.$$

Use the Sylvester's expansion to determine the state transition matrix and compute the force-free evolution from $\mathbf{x}(0) = (1, 2)^T$.