November 15, 2018

CK0255/TIP7244: Assignment A

Exercise 01 (20%). Consider the following IO representation of a system

$$\frac{\mathrm{d}^3}{\mathrm{d}t^3}y(t) + (1-\varrho)t^2\frac{\mathrm{d}^2}{\mathrm{d}t^2}y(t) + \frac{\mathrm{d}}{\mathrm{d}t}y(t) + y^{\eta}(t) = \frac{\mathrm{d}^2}{\mathrm{d}t^2}u(t) + 2\frac{\mathrm{d}}{\mathrm{d}t}u(t),$$

with $\varrho \in \mathcal{R}$ and $\eta \in \mathcal{R}$ unknown constant parameters.

A. Discuss the properties of the system (linear/non-linear, stationary/non-stationary, dynamic/instantaneous and causal/non-causal) for all possible values of ϱ and η. $\rightsquigarrow$ Justify your answers.

Exercise 02 (20%). Consider the following SS representation of a system/model

$$\begin{cases} \dot{x}_1(t) &= \alpha x_1(t) - \beta t^2 x_2(t) + 5u(t) \\ \dot{x}_2(t) &= \beta e^t x_1(t) + \alpha x_2^3(t) + u(t-5) \\ y(t) &= x_1(t) + x_2(t) + (1-\alpha)u(t) \end{cases},$$

with $\alpha \in \mathcal{R}$ and $\beta \in \mathcal{R}$ unknown constant parameters.

A. Discuss the properties of the system (linear/non-linear, stationary/non-stationary, dynamic/instantaneous and causal/non-causal) for all possible values of ϱ and η. $\rightsquigarrow$ Justify your answers.

Exercise 03 (30%). Consider the following IO representation of a system

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}y(t) + 2\frac{\mathrm{d}}{\mathrm{d}t}y(t) + \eta y(t) = 3u(t),$$

with $\eta \in \{17, 65\}$ a constant parameter that can take on two known values.

A. Show that the system's evolution is characterised by a pseudo-periodic mode, for both values of η. $\rightsquigarrow$ Justify your answers.
B. Determine the roots of the characteristic polynomial in the complex plane and determine the system's parameters (time constant τ, natural frequency ω_n and dumping factor ξ), for both values of η. $\rightsquigarrow$ Justify your answers.
C. Discuss the influence of η on ξ and τ and sketch the system's modes.

Exercise 04 (30%). Consider the following SS representation of a system

$$\begin{cases} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u(t) \\ \\ y(t) = \begin{bmatrix} 0 & 3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} u(t) \end{cases}$$

A. Determine the dimensions of the input-, output and state-vectors and determine the eigenvectors and eigenvalues of matrix $\mathbf{A}$. $\rightsquigarrow$ Justify your answers.
B. Discuss the existence of a similarity transformation $\mathbf{x}(t) = \mathbf{P}\mathbf{z}(t)$ such that the new representation is characterised by a diagonal state matrix. Determine such a diagonal representation. $\rightsquigarrow$ Justify your answers.
C. Determine the state transition matrix for both the original and the diagonal representation. $\rightsquigarrow$ Justify your answers.