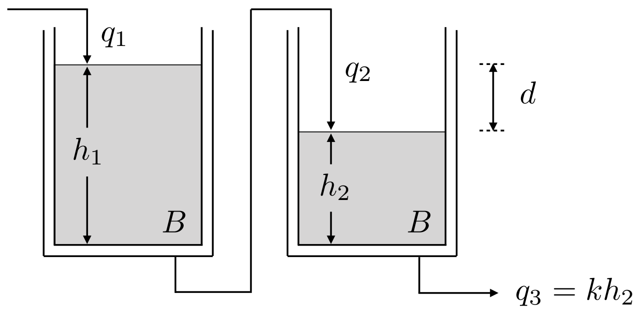
CHEM-E7190/2020: Exercise I

Task 1. Consider the two-tank system in the figure below. Let the feed flow-rates $q_1(t)$ and $q_2(t)$ to the two tanks be the control inputs ($u_1(t)$ and $u_2(t)$, in control notation), and let the volumes $V_1(t)$ and $V_2(t)$ of liquid in the two tanks be the state variables ($x_1(t)$ and $x_2(t)$, in control notation).



Assume that we cannot directly measure the liquid volumes in the tanks, only their difference in level. Let $d(t) = V_1(t)/B - V_2(t)/B$ be the output variable ($y_1(t)$, in control notation). Also assume that the feed flow-rate from the second tank is proportional to the liquid level in that tank and that it is measured, $q_3(t) = kV_2(t)/B$ (equivalently, $y_2(t) = k/Bx_2(t)$).

We can write the total mass balance equations (the state equations)

$$\begin{cases} \dfrac{\mathrm{d}V_1(t)}{\mathrm{dt}} = q_1(t) - q_2(t) \\ \dfrac{\mathrm{d}V_2(t)}{\mathrm{dt}} = q_2(t) - (k/B)V_2(t) \end{cases} \quad \rightsquigarrow \quad \underbrace{\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix}}_{\dot{x}(t)} = \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & -k/B \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}}_{x(t)} + \underbrace{\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}}_{u(t)}$$

Similarly, we can write the measurement (output) equations

$$\begin{cases} d(t) = (V_1(t)) - V_2(t))/B \\ q_3(t) = (k/B)V_2(t) \end{cases} \quad \rightsquigarrow \quad \underbrace{\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}}_{y(t)} = \underbrace{\begin{bmatrix} 1/B & -1/B \\ 0 & k/B \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}}_{x(t)} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}}_{u(t)}$$

Check model validity and familiarise with programs `twoTanks_main.m` and `twoTanks.m` (0.). Experiment on how to simulate the system from different initial conditions $x(t=0) = \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix}$ and varying inputs $u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$.

1. Simulate the system from initial condition $x(0) = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ for a constant input $u(t) = \begin{bmatrix} 0.1 \\ 2 \end{bmatrix}$, $t \in [0, 12.5]$.
   - Does the system reach a steady-state? If positive, what are the steady-state values (u_{SS}, x_{SS})?
   - If negative, what should be done to observe the system reach a steady-state?
2. Simulate the system from initial condition $x(0) = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ for an input defined as follows
   - $u_1(t) = 1$ for $t \in [0, 5)$, $u_1(t) = 0.5$ for $t \in [5, 10)$, and $u_1(t) = 2$ for $t \in [10, 15]$;
   - $u_2(t) = 0.5$ for $t \in [0, 2.5)$, $u_2(t) = 1$ for $t \in [2.5, 10)$, $u_1(t) = 2.5$ for $t \in [10, 15]$.
3. Simulate the system with the input defined above and initial condition $x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

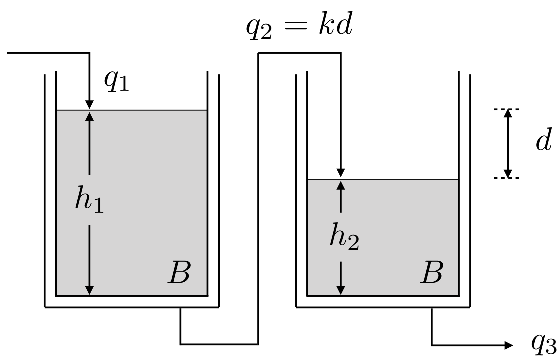
A more realistic assumption for the effluent flow-rate q_3 would be to let $q_3(t) = k\sqrt{h_2(t)}$ (Torricelli's law).

4. What are the implications of such modelling assumption?

5. Simulate the resulting model to test your considerations.

You can use program `plotTwotanks.m` to plot your results. You can also modify it to suit your needs.

Task 2. Consider a modified two-tank system, in the figure below. Study the process diagram to understand the changes, then write state and measurement equations, using both process and control notation (0.).



Use the template programs for Task 1 to create your `twoTanks_flipped_main.m` and `twoTanks_flipped.m` codes and test them when simulating the system from various initial conditions $x(0)$ and varying inputs $u(t)$.

1. Simulate the system from initial condition $x(0) = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ for a constant input $u(t) = \begin{bmatrix} 0.1 \\ 2 \end{bmatrix}$, $t \in [0, 12.5]$.
   - Does the system reach a steady-state? If positive, what are the steady-state values (u_{SS}, x_{SS})?
   - If negative, what should be done to observe the system reach a steady-state?

2. Simulate the system from initial condition $x(0) = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ for an input defined as follows
   - $u_1(t) = 1$ for $t \in [0, 5)$, $u_1(t) = 0.5$ for $t \in [5, 10)$, and $u_1(t) = 2$ for $t \in [10, 15]$;
   - $u_2(t) = 0.5$ for $t \in [0, 2.5)$, $u_2(t) = 1$ for $t \in [2.5, 10)$, $u_1(t) = 2.5$ for $t \in [10, 15]$.

3. Simulate the system with the input defined above and initial condition $x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

A more realistic assumption for the internal flow-rate q_2 would be to let $q_2(t) = k\sqrt{d(t)}$.

4. What are the implications of such modelling assumption?

5. Simulate the resulting model to test your considerations.

Modify program `plotTwotanks.m` to create `plotTwotanks_flipped.m`. Use your code to plot the results.