Exercise 1. Consider a continuous stirred tank reactor in which the reaction scheme occurs


$$
\begin{array}{r}
A \xrightarrow{k_{1}} B \xrightarrow{k_{2}} C \\
2 A \xrightarrow{k_{3}} D \tag{1b}
\end{array}
$$

Component $B$ is the desired product and we assume that we can measure its composition in the reactor, $[B](t)$. We also assume that the feed only contains component $A$, whose composition $[A]_{i}(t)$ can be set, and that density, temperature and volume in the reactor are constant.

Let $F_{i}(t)\left[\mathrm{lt} \mathrm{min}{ }^{-1}\right]$ be the volumetric flow-rate of the inlet stream, $F_{o}(t)\left[\mathrm{lt} \mathrm{min}{ }^{-1}\right]$ the volumetric flow-rate of the outlet stream, and let $F^{S S} / V=4 / 7\left[\mathrm{~min}^{-1}\right]$ be the dilution-rate/space-velocity at a steady-state operation point, $V[\mathrm{lt}]$ indicates the volume. Let $[A]_{i}^{S S}=10\left[\mathrm{~mol} \mathrm{lt}^{-1}\right]$ be the concentration of component $A$ in the feed at that steadystate. The rate constants are i) $k_{1}=5 / 6\left[\mathrm{~min}^{-1}\right]$; ii) $k_{2}=5 / 3\left[\mathrm{~min}^{-1}\right]$; and, iii) $k_{3}=$ $1 / 6\left[\mathrm{~mol} \mathrm{lt}^{-1} \mathrm{~min}^{-1}\right]$, with $A \rightarrow B$ and $B \rightarrow C$ characterised by first-order rates of reaction per unit volume and $A+A \rightarrow D$ characterised by a second-order rate per unit volume.

- Indicate input, output and state variables. Comment on their properties (measurable, manipulable, controllable, control, disturbance, ...);
- Write the total material balance and the material balances for all of the components;
- Write the complete state-space model of this system. Use the control notation for state, input and output variables, and parameters, to replace the problem-specific quantities;
- Given the steady-state values of the dilution rate and feed composition, determine the steady-state concentration of component $A,[A]^{s s}$, and use it to determine the steady states concentrations of the components $B, C$ and $D$ (that is, $[B]^{S S},[C]^{S S}$, and $[D]^{S S}$ );
- Plot the function $[B]^{S S}=h\left(F^{S S} / V\right)$ and comment on the choice of $F^{S S} / V=4 / 7\left[\mathrm{~min}^{-1}\right]$ as operating point for this reactor;
- Linearise the state-space model and compute the specific model realisation $(A, B, C, D)$ corresponding to the steady-state operating point that you have calculated;
- Compute the eigenvalues and eigenvectors of the state matrix $A$ and comment on the stability of the reactor under the assume linear approximation.

