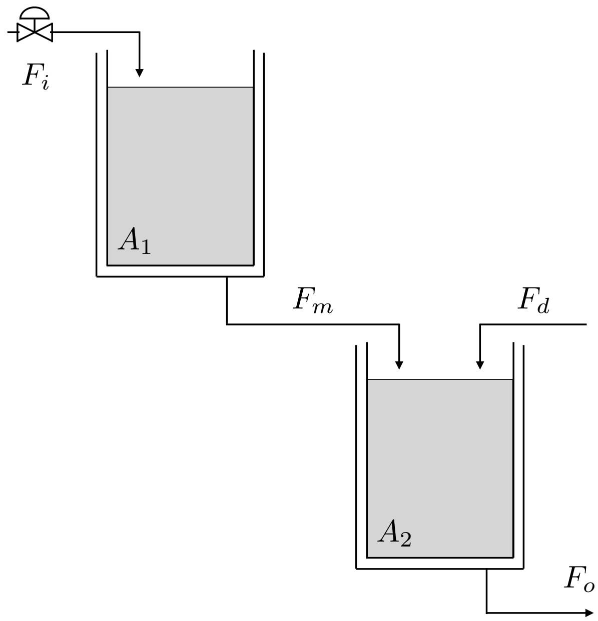
CHEM-E7140 (Exam)

Exercise 1. Consider a pair of surge tanks of cross-sectional area A_1 and A_2, respectively.



The tanks, depicted in the figure, are drained by gravity in such a way that the outflow from the first tank is $F_m(t) = \alpha_1(h_1)^{1/2}$ and the outflow from the second tank is $F_o(t) = \alpha_2(h_2)^{1/2}$; α_1 and α_2 indicate the resistance-to-flow coefficients associated with these outlet streams, $h_1(t)$ and $h_2(t)$ are liquid levels in the tanks Moreover, denote the influent flow rate to the first tank as $F_i(t)$ and we let $F_d(t)$ indicate an additional influent to the second tank.

We assume that liquid levels $h_1(t)$ and $h_2(t)$ are measured and that also $F_d(t)$ is measured.

1. Write down the total mass balance for the system (10%);

2. Identify state-variables, input variables, and measured variables (10%);

3. Rewrite the total mass balance as state-space model in terms of x, u and y (10%);

4. Assuming steady-state conditions $\widetilde{F}_i = \widetilde{F}_i$, $\widetilde{F}_d = 0$, $\widetilde{h}_1 = \widetilde{F}_i/\alpha_1^2$ and $\widetilde{h}_2 = \widetilde{F}_i/\alpha_2^2$, linearise the state-space model around this fixed point and write it down in terms of deviation variables x', u' and y' (40%);

5. For $A_1 = 1$, $A_2 = 1$, $\alpha_1 = 1$, $\alpha_2 = 1$ and $\widetilde{F}_i = 1$, *i*) study the stability of the linearised state-space model (10%); *ii*) compute the controllability matrix and comment on the controllability of the pair (A, B) (10%); and, *iii*) compute the observability matrix of the system and comment on the observability of the pair (A, C) (10%).