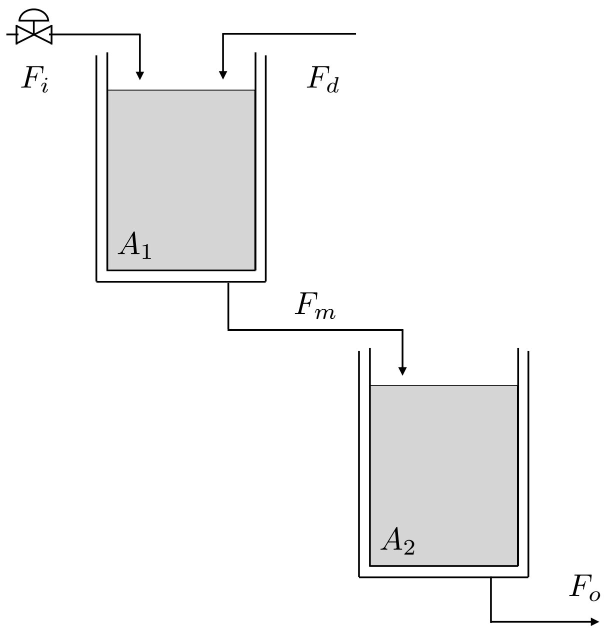
Exercise 1. Consider a pair of surge tanks of cross-sectional area A_1 and A_2, respectively.



The tanks, depicted in the figure, are drained by gravity in such a way that the outflow from the first tank is $F_m(t) = \alpha_1(h_1)^{1/2}$ and the outflow from the second tank is $F_o(t) = \alpha_2(h_2)^{1/2}$; α_1 and α_2 indicate the resistance-to-flow coefficients associated with these two streams, $h_1(t)$ and $h_2(t)$ indicate the liquid levels in the tanks Moreover, we denote the influent flow-rate to the first tank as $F_i(t)$ and we let $F_d(t)$ indicate an additional influent to the first tank.

The control task consists of controlling the liquid levels in the two tanks by manipulating the influent flow-rate to the first tank, when the flow-rate of the additional influent to the first tank is zero. We assume that the liquid levels $h_1(t)$ and $h_2(t)$ in the two tanks are measured and we assume that all the flow-rates ($F_i(t)$, $F_m(t)$, $F_d(t)$, and $F_o(t)$) are measured, too.

1. Derive the total mass balances for the two tanks and treat it as system model (10%);
2. Identify state-variables, input variables, and measured variables (10%);
3. Rewrite the total mass balance as state-space model in terms of x, u and y (10%);
4. Assuming steady-state conditions $\widetilde{F}_i$, $\widetilde{h}_1 = \widetilde{F}_i/\alpha_1^2$ and $\widetilde{h}_2 = \widetilde{F}_i/\alpha_2^2$, linearise the state-space model around this fixed point and write it down in linearised form using the deviation variables x', u' and y' (40%);
5. For $A_1 = 1$, $A_2 = 1$, $\alpha_1 = 1$, $\alpha_2 = 1$ and $\widetilde{F}_i = 1$, *i*) study the stability of the linearised state-space model (10%); *ii*) compute the controllability matrix and comment on the controllability of the pair (A, B) (10%); and, *iii*) compute the observability matrix of the system and comment on the observability of the pair (A, C) (10%).