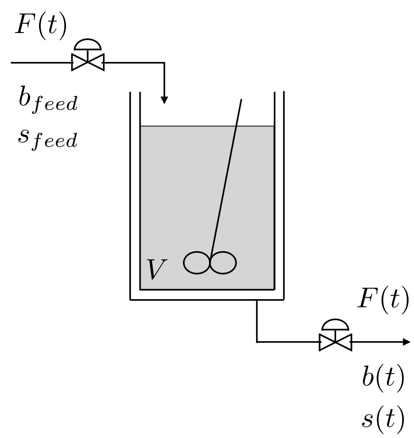
Exercise Q1. Biochemical reactors are process units used in the production of a wide variety of intermediate and final products, including food, beverages, and pharmaceuticals. Similarly to chemical reactors their models are based on material balances. The simplest bioreactors consists of two components: biomass and substrate. The biomass consists of cells that consume substrate, as in fermentation where cells consume sugar and produce alcohol.



Consider a perfectly mixed and continuous biochemical reactor with constant liquid volume V (litres), see figure. Let $b(t)$ ands $s(t)$ be the biomass and substrate concentrations in time (t), respectively: That is, $b(t) = \dfrac{\text{mass of cells at } t}{\text{volume}}$ and $s(t) = \dfrac{\text{mass of substrate at } t}{\text{volume}}$. Let $b_{feed} = 0$ and $s_{feed} = 4$ be the feed concentrations (in grams per litre), we shall assume them to be constant in time. Let $F(t)$ be the volumetric feed flow-rate (in litres per hour).

(**Q1.1**) Write the material balances for biomass and substrate assuming the following:

⇝ Instantaneous (at t) rate of biomass generation

$$\underbrace{r_b(t)}_{+} = \frac{\text{mass of cells } \underline{\underline{\text{generated}}}}{\text{volume} \times \text{time}} \tag{1a}$$

$$= \mu(t)b(t) \tag{1b}$$

⇝ Instantaneous (at t) rate of substrate consumption

$$\underbrace{r_s(t)}_{-} = \frac{\text{mass of substrate } \underline{\underline{\text{consumed}}}}{\text{volume} \times \text{time}} \tag{2a}$$

$$= 0.4 \times (\mu(t)b(t)) \tag{2b}$$

where the growth rate coefficient $\mu(t)$ is assumed to be of the Monod type: That is,

$$\mu(t) = \mu_{\max}\frac{s(t)}{k_{\mathrm{m}} + s(t)}, \qquad \text{with } \begin{cases} \mu_{\max} = 0.53 \left[\ \mathrm{hr}^{-1}\right] \\ k_{\mathrm{m}} = 0.12 \left[\ \mathrm{g\ l}^{-1}\right] \end{cases} .$$

(**Q1.2**) Assuming that we measure the substrate concentration, list the state variables (x) and their dimensionality (N_x), the control variables (u) and their dimensionality (N_u), the output variables y and their dimensionality (N_y), and the model parameters (θ_x and θ_y). Then, write the state-space model (dynamics and measurement equations) for this problem.

(**Q1.3**) Assume i) a steady-state dilution rate $D_{\text{ss}} = F_{\text{ss}}/V = 0.15 \left[\text{ hr}^{-1}\right]$ associated to a steady-state feed flow-rate $u_{\text{ss}} = F_{\text{ss}}$ and a volume V of your choice; and ii) a steady state $x_{ss} = (b_{\text{ss}} = 1.5374, s_{\text{ss}} = 0.1565)$, then determine the linear time-invariant (LTI) approximation of the nonlinear state-space model: That is, determine the matrices $(A, B, C.D)$ from the Jacobians of dynamics and measurement equation evaluated at the equilibrium $(x_{\text{ss}}, u_{\text{ss}})$.

Exercise Q2. The linearisation of dynamics and measurement equation at a steady-state point (x_{SS}, u_{SS}) of a process led to the linear time-invariant (LTI) state-space model below:

$$\dot{x}(t) = \begin{bmatrix} 1 & -3 \\ 0 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t) \tag{3a}$$

$$y(t) = \begin{bmatrix} 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \end{bmatrix} u(t) \tag{3b}$$

(**Q2.1**) Determine the stability of the steady-state point / system.

(**Q2.2**) Determine the controllability matrix of the system (Q2.2a), then use it to establish whether the system is controllable or not (Q2.2b).

(**Q2.3**) Determine the observability matrix of the system (Q2.3a), then use it to establish whether the system is controllable or not (Q2.3b).

Give-me-points table:

- **Q1**: Max 65/100 points
  - **Q1.1**: 20/100 points
  - **Q1.2**: 15/100 points
  - **Q1.3**: 30/100 points
- **Q2**: Max 35/100 points
  - **Q2.1**: 05/100 points
  - **Q2.2**: 15/100 points
  - **Q2.3**: 15/100 points

This is an open-book examination. In addition to pencil/pen, eraser and other writing material, the use of own printed copies of the course material and personal notes is allowed.