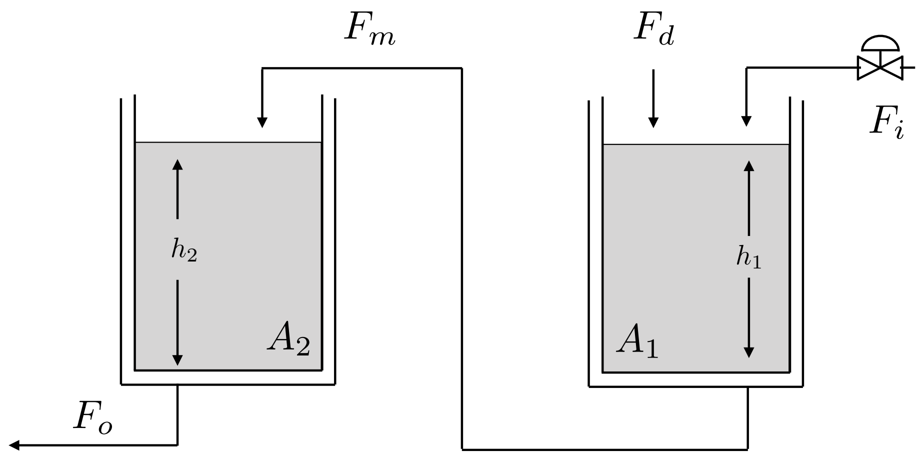
Exercise 1. Consider two cylindric reservoirs used for storing lwater. The geometry of the vessel is summarised by the cross-sectional areas A_1 and A_2 and the levels $h_1(t)$ and $h_2(t)$ which indicate the liquid hold-up at time t.

We will assume that water is extracted from the vessels exclusively by gravity: That is, no pumps are used. The outflow from the first tank ito the second tank is $F_m(t) = \alpha_1 h_1(t)^{1/2}$, whereas the outflow from the second tank is $F_o(t) = \alpha_2 h_2(t)^{1/2}$. α_1 and α_2 are constant resistance-to-flow coefficients. For the sake of completeness, we will denote the influent flow-rate to the first tank as $F_i(t)$ and we let $F_d(t)$ be an additional influent to the first tank.



We are interested in controlling the level of water in the two tanks by manipulating the influent flow-rate F_i to the first tank. Moreover, we will assume that both $h_1(t)$ and $h_2(t)$ are measured quantities and that $F_i(t)$, $F_m(t)$, $F_d(t)$, and $F_o(t)$ are measured, as well.

1. Write the total mass balances for the two vessels and use it as system model (10%);
2. Determine which process variables are input, measured, and state variables (10%);
3. Restate the total mass balance in state-space form in terms of x, u and y (10%);
4. Consider the steady-state $\left(\widetilde{F}_i, \widetilde{h}_1 = \widetilde{F}_i \alpha_1^{-2}, \widetilde{h}_2 = \widetilde{F}_i \alpha_2^{-2}, F_d = 0\right)$, linearise the model around it and write its approximation using the deviation variables x', u' and y' (30%);
5. For $A_1 = 1$, $A_2 = 1$, $\alpha_1 = 1$, $\alpha_2 = 1$ and $\widetilde{F}_i = 1$, *i)* study the stability of the linear approximation (10%); *ii)* compute its controllability matrix and comment on the full-state controllability of the model pair (A, B) (10%); and, *iii)* compute its observability matrix and comment on the full-state observability of the model pair (A, C) (10%).

This is an open-book examination. In addition to pencil/pen, eraser and other writing material, the use of own printed copies of the course material and personal notes is allowed.