

[February 23, 2021]

CHEM-E7190 (Exam)

Exercise 01. The linearisation of a state-space and instrument process model around some steady-state operating point (x_{SS}, u_{SS}) leads to the following linear and time-invariant (LTI) dynamics and measurement equations:

$$\dot{x}(t) = \begin{bmatrix} 1 & -6 \\ 0 & -5 \end{bmatrix} x(t) + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} u(t) \quad (1a)$$

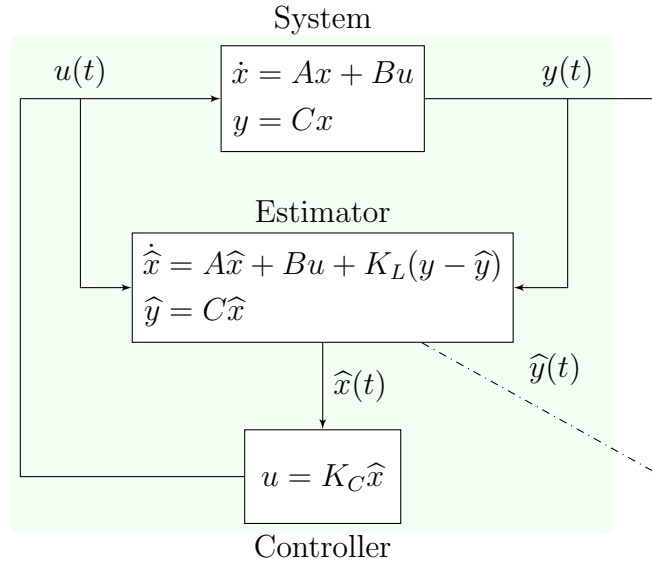
$$y(t) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} x(t) \quad (1b)$$

- (10%) Briefly define what is the meaning of Eq. (1a) and Eq. (1b). For each equation, use simple words and max two short sentences;
- (10%) Briefly define what the process variables $\dot{x}(t)$, $x(t)$, $u(t)$, and $y(t)$ are and determine how many (their dimension) of each are used in Eq. (1). For each variable, use simple words and max two short sentences;
- (10%) Identify all the matrices involved in the model (state-state, input-state, state-output, and input-output matrix) and determine their size.

Exercise 02. We are interested in controlling the process system above using a state-feedback approach. For the task, we must firstly verify that the system is both controllable and observable and then design a state feedback controller and a state observer that operate together with the system.

- (10%) Define the notion of controllability and what a feedback controller does. Use simple words and max three short sentences;
- (10%) Define the notion of observability and what a state observer does. Use simple words and max three short sentences;
- (10%) Define the controllability matrix and compute it for the system in Eq. (1). Can you determine from the controllability matrix that you computed whether the system is controllable?
- (10%) Define the observability matrix and compute it for the system in Eq. (1). Can you determine from the observability matrix that you computed whether the system is observable?

The block-diagram representing the system-observer-controller is below



The observer is given by

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + K_L(y(t) - \hat{y}(t)); \quad (2a)$$

$$\hat{y}(t) = C\hat{x}(t). \quad (2b)$$

The controller is given by

$$u(t) = K_C \hat{x}(t). \quad (3)$$

- (10%) Briefly define what is the meaning of Eq. (2a), Eq. (2b), and Eq. (3). For each equation, use simple words and max four short sentences;
- (10%) Briefly define what the process variables $\hat{x}(t)$, $\hat{y}(t)$ are and determine how many (their dimension) of each are used in the model in Eq. (1). For each variable, use simple words and max two short sentences;
- (10%) Define all new matrices involved (K_L and K_C) and determine their size.