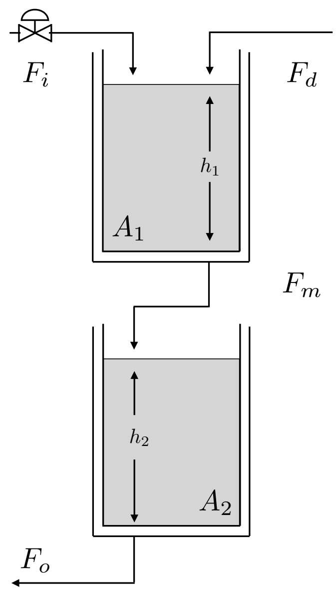
[February 2023] **CHEM-E7190**

Exercise 1. Consider two surge tanks for storing liquids. The tanks have cross-sectional area A_1 and A_2 and $h_1(t)$ and $h_2(t)$ indicate the liquid levels in the tanks at time t. The tanks are emptied by gravity: The outflow from the first tank is $F_m(t) = \alpha_1\sqrt{h_1(t)}$ and the outflow from the second tank is $F_o(t) = \alpha_2\sqrt{h_2(t)}$, with α_1 and α_2 the resistance-to-flow coefficients associated with the outflow streams. Moreover, we denote the influent flow-rate to the first tank as $F_i(t)$ and we let $F_d(t)$ indicate an additional influent to the first tank.



We are interested in controlling the liquid level in the two tanks and we want to use the influent flow-rate F_i to the first tank as manipulated variable. We assume that $h_1(t)$ and $h_2(t)$ are measured and we assume that $F_i(t)$, $F_m(t)$, $F_d(t)$, and $F_o(t)$ are measured, too.

1. Derive the total mass balances for the two tanks and treat it as system model (10%);

2. Identify input variables, measured variables, and state variables (10%);

3. Rewrite the total mass balance as state-space model in terms of x, u and y (10%);

4. Assume the steady-state conditions $\widetilde{F}_i$, $\widetilde{h}_1 = \widetilde{F}_i\alpha_1^{-2}$, $\widetilde{h}_2 = \widetilde{F}_i\alpha_2^{-2}$, and $F_d = 0$. Linearise the model around this fixed point and write its linear approximation using the deviation variables x', u' and y' (30%);

5. For $A_1 = 1$, $A_2 = 1$, $\alpha_1 = 1$, $\alpha_2 = 1$ and $\widetilde{F}_i = 1$, i) study the stability of the linearised state-space model (10%); ii) compute the controllability matrix and comment on the controllability of the pair (A, B) (10%); and, iii) compute the observability matrix of the system and comment on the observability of the pair (A, C) (30%).

This is an open-book examination. In addition to pencil/pen, eraser and other writing material, the use of own printed copies of the course material and personal notes is allowed.