CK0030: Partial evaluation II

Exercise 01 (50%).

Consider the function $y(x) = x + \sin(x) + 2\cos(2x) - 0.00004e^{(x)}$, $x \in \mathbb{R}$.

Write a program based on `FOR-loops` that implements the approximation

$$y(x) \approx x + \sum_{k_1=0}^{N_1} \frac{(-1)^{k_1} x^{(2k_1+1)}}{(2k_1+1)!} + 2\sum_{k_2=0}^{N_2} \frac{(-1)^{k_2}(2x)^{2k_2}}{(2k_2)!} - 0.00004 \sum_{k_3=0}^{N_3} \frac{x^{k_3}}{k_3!}$$

Let $x = 6$ and use $N_1 = 16$, $N_2 = 16$ and $N_3 = 32$.

Solution:

```
1 from math import factorial as fact
2
3 x = [6.]; nx = len(x)                              # x = [1.,2.,3.,4.,5.,6.,7.]
4
5 k2_max = 16
6 k3_max = 16
7 k4_max = 32
8
9 s1 = nx*[0]
10 s2 = nx*[0]
11 s3 = nx*[0]
12 s4 = nx*[0]
13
14 y   = nx*[0]
15
16 for ix in range(0,nx):
17
18  s1[ix] = x[ix]
19
20  s2[ix] = 0.
21  for k2 in range(0,k2_max+1):
22   t2_num = (-1)**k2 * x[ix]**(2*k2+1)
23   t2_den = fact(2*k2+1)
24   t2 = t2_num/t2_den
25   s2[ix] = s2[ix] + t2
26
27  s3[ix] = 0.
28  for k3 in range(0,k3_max+1):
29   t3_num = (-1)**k3 * (2*x[ix])**(2*k3)
30   t3_den = fact(2*k3)
31   t3 = t3_num/t3_den
32   s3[ix] = s3[ix] + t3
33
34  s4[ix] = 0.
35  for k4 in range(0,k4_max+1):
36   t4 = (x[ix]**k4)/fact(k4)
37   s4[ix] = s4[ix] + t4
38
39  y[ix] = s1[ix] + s2[ix] + 2*s3[ix] - 0.00004*s4[ix]
```

Exercise 02 (50%).

Consider the following Python program

```
1  z  =  3**(-1)*n*(n+1)*(n+2)
2
3  n  =  32
4  s  =  0
5  for  i  in  range(0,n+1):
6    for  j  in  range(0,i):
7      for  k  in  range(j,i+(j+1)):
8        s  +=  1
```

The code is a bad implementation of the following identity

$$\sum_{i=1}^{n} \left(\sum_{j=1}^{i} \left(\sum_{k=j}^{i+j} 1 \right) \right) = (n+2)(n+1)(n+0)/3$$

Find and correct the potential mistakes that are in the code.

Solution:

```
1  n=32                                        # 1 (before z)
2  z  =  3**(-1)*n*(n+1)*(n+2)
3
4  s=0
5  for  i  in  range(1,n+1):                   # 2 (start at 1)
6    for  j  in  range(1,i+1):                 # 3, 4 (start at 1, end at i)
7      for  k  in  range(j,i+(j+1)):
8        s  +=  1
```