May 30, 2019

CK0030: Partial evaluation II

Exercise 01.

Let $\mathcal{A} = \{0.0001, 0.001, 0.01, 0.1, 1\}$ and let $\mathcal{B} = \{0.0001, 0.001, 0.01, 0.1, 1\}$.

Write Python code that computes for $N = 128$ the expression

$$\sum_{n=0}^{N} \frac{1}{n!} \sum_{m=0}^{n} \frac{n!}{m!(n-m)!} a^m b^{n-m}$$

for each pair (a, b), with $a \in \mathcal{A}$ and $b \in \mathcal{B}$.

Comment and explain your solution/code.

Solution:

$$\exp(a+b) \approx \overbrace{\sum_{n=0}^{N} \Big\{ \underbrace{\frac{1}{n!} \Big[\overbrace{\sum_{m=0}^{n} \underbrace{\frac{n!}{m!(n-m)!} a^m b^{n-m}}_{\texttt{t\_int}}}^{\texttt{s\_int}} \Big]}_{\texttt{t\_ext}} \Big\}}^{\texttt{s\_ext}}$$

```
1  from math import factorial, exp
2
3  A = [0.0001, 0.001, 0.01, 0.1, 1]                          # Define set A
4  B = [0.0001, 0.001, 0.01, 0.1, 1]                          # Define set B
5
6  N = 128                                                    # Define N
7  for a in A:                                                # For each a in A
8    for b in B:                                              # For each b in B
9
10     s_ext = 0                          # Set external sum to be equal to zero
11         n = 0                        # Set external index to be equal to zero
12         while n <= N:
13             t_ext = 1/factorial(n)              # External term, first part
14
15             s_int = 0                # Set internal sum to be equal to zero
16             m = 0                  # Set internal index to be equal to zero
17             while m <= n:
18                 t_int = factorial(n) * a**m * b**(n-m) \   # Internal term
19                         / factorial(m) / factorial(n-m)    #
20
21                 s_int = s_int + t_int                            # Update sum
22                 m = m + 1                              # Update internal index
23
24             t_ext = t_ext * s_int                     # Finish external term
25
26             s_ext = s_ext + t_ext                       # Update external sum
27             n = n + 1                                 # Update external index
28
29         print(a,b,s_ext,exp(a+b))                         # Print and compare
```