Approximations to the solutions of the filtering equations for the stochastic chemostat

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Abstract:
Chemostats are used to study the growth and the harvesting of desired cells. These cultivation devices allow for the study of the relationship between fitness, protein expression and metabolic fluxes, as well as the study of the cooperation and competition between multiple species. Characterising the dynamics in these systems is essential for devising strategies for optimal operation while sustaining the delicate conditions required for life of the species. In order to characterise the dynamics, we consider a differential description of the evolution of the state of a chemostat under environmental fluctuations. We present solutions to evolution equations for a system subjected to geometric Brownian motion. Under this modelling assumption, our best knowledge about the state is given by its distribution in time, given the initial state. Such a function solves a deterministic partial differential equation, the Kolmogorov forward equation. In this work, however, we refine our knowledge about the state of the chemostat when information about the system is available in the form of easy-to-acquire measurements. More formally, we are interested in obtaining the solution to a non-linear stochastic partial integral differential equation, the Kushner-Stratonovich equation. By solving this equation, we can obtain the distribution of the state conditional on measurements. For the chemostat, this solution is not available in closed form, and it must be approximated. We present an approximation to the solution to the Kushner-Stratonovich equation based on methods for partial differential equations. We compare the obtained solution with the approximation via sequential Monte Carlo methods, known as particle filters, and with a commonly used linear approximation of the dynamics, known as the Kalman-Bucy filter.

Keywords: state estimation, Fokker-Planck equation, Kushner-Stratonovich equation, stochastic chemostat

(To self) Supporting material for introductory sentences:

Fitness, protein expression and metabolic fluxes

Competition and cooperation