A System Level Approach to Closed-loop Best-Response Dynamics

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Modern cyber-physical systems are usually comprised of multiple subsystems operated by local self-interested decision-making agents. The inherent large-scale and distributed nature of most applications, together with information asymmetry, render the centralized approach to controller design not applicable. Dynamic game theory provides a framework for determining competitive equilibria (e.g., the Nash equilibrium [1, 2]) which provide locally optimal, yet strategically stable, operating conditions for each non-cooperative agent. However, the computation of a Nash equilibrium is a challenging task, except for specific problems [3]. In particular, potential games characterise a broad class of problems for which a Nash equilibrium can be obtained efficiently using Best-Response Dynamics (BRD) algorithms [4, 5]. In dynamical settings, BRD methods are mostly studied for computing open-loop Nash equilibria, where the strategy of each player depends only on the initial state of the game. The design of numerical routines for obtaining closed-loop equilibria, where agents react to changes in the game state, is still under active research.

In this work, we present a method towards the computation of closed-loop ε-Nash equilibria of dynamic potential games. We restrict ourselves to affine-quadratic difference games. Leveraging the recent System Level Synthesis approach (SLS, [6]), we propose a BRD algorithm in which the agents iteratively update their strategies by optimizing system responses from the other players' actions. By parameterizing all stabilizing controllers, these system responses can then be used to reconstruct a state-feedback control strategy for each player. Due to its closed-loop nature, the resulting strategy profiles allow the agents to simultaneously react to perturbations on the game state. Additionally, the method allows to enforce desirable structures to each controller (e.g., spatiotemporal information patterns), without changing the structure of the game. We demonstrate the behaviour of this method on illustrative examples.

References

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