Matrícula (First-name Last-name)

CK0191: Homework A

Exercise A.1. For a given function $f(x)$, the integral $\int_a^b f(x)\mathrm{d}x$ computed using the formula

$$\int_a^b f(x)\mathrm{d}x \approx h\Big[\frac{1}{2}f(x_0) + \sum_{i=1}^{n-1} f(x_i) + \frac{1}{2}f(x_n)\Big], \qquad (1)$$

is approximated by n trapezoids of equal width h.

Write a Python function that takes any f, a and b, and n as inputs and returns the approximation.

Solution: We write a Python function `trapz.py` with variables corresponding to the notation

```
def trapz(f, a, b, n):
    h = float(b-a)/n
    result = 0.5*f(a) + 0.5*f(b)  # 1st and 3rd term between brackets
    for i in range(1, n):
        result += f(a + i*h)      # Loop through index i (2nd term)
    result *= h                   # Final multiplication
    return result
```

The function can be tested as follows

```
>>> from trapz import trapz
>>> from math import exp
>>> v = lambda t: 3*(t**2)*exp(t**3)
>>> n = 4
>>> num_int = trapz(v, 0, 1, n)
>>> num_int
    1.9227167504675762
```

Exercise A.2. Consider a system with SS representation given by a linear and stationary model

$$\begin{cases} \dot{x}(t) & = Ax(t) + Bu(t) \\ y(t) & = Cx(t) \end{cases}.$$

Let $A = \begin{bmatrix} 0 & 1 \\ -4 & -4 \end{bmatrix}$, $B = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 1 \end{bmatrix}$.

Determine the Sylvester expansion of the state transition matrix e^{At}, show the modes of the system.

Solution:

$$e^{At} = \begin{bmatrix} e^{-2t} + 2te^{-2t} & te^{-2t} \\ -4te^{-2t} & e^{-2t} - 2te^{-2t} \end{bmatrix}.$$