Exercise 01. Consider a linear and stationary system in SS representation

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$
$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)$$

Consider the following system matrices

$$\mathbf{A} = \begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}; \quad \mathbf{C} = \begin{bmatrix} 0 & 3 \end{bmatrix}; \quad \mathbf{D} = \begin{bmatrix} 0 \end{bmatrix}$$

1. Compute the eigenvalues and the eigenvectors of matrix $\mathbf{A}$;

2. Determine the modes of matrix $\mathbf{A}$ and plot their time evolution;

3. Use the Sylvester expansion to compute the state transition matrix;

4. Discuss the existence of a similarity transformation $\mathbf{x}(t) = \mathbf{P}\mathbf{z}(t)$ that leads to a diagonal representation of the state matrix. Show the resulting state matrix, if it exists.

Exercise 02. Consider a linear and stationary system in SS representation

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$
$$y(t) = \begin{bmatrix} 2 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

1. Use the Sylvester expansion to determine the state transition matrix;

2. Determine a similarity transformation that leads to a state matrix in diagonal form. Show the resulting state matrix, if it exists.