Exercise 01. Consider the linear time-invariant system given in SS representation

$$\begin{aligned}\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} &= \begin{bmatrix}-3 & 1\\ 1 & -3\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + \begin{bmatrix}1\\ -1\end{bmatrix}u(t)\\ y(t) &= \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}\end{aligned}.$$

- Determine a new representation in which the new state variables are

$$\begin{aligned}z_1(t) &= x_1(t) + x_2(t)\\ z_2(t) &= x_1(t) - x_2(t)\end{aligned};$$

- Determine the corresponding similarity transformation $\mathbf{z} = \mathbf{P}^{-1}\mathbf{x}$ and calculate all the system matrices in the new representation;
- In the original representation, let $\mathbf{x}(0) = (4, 2)^T$. Determine the state transition matrix and the force-free evolution of the system from $\mathbf{x}(0)$.

Exercise 02. Consider the linear time-invariant system given in SS representation

$$\begin{aligned}\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} &= \begin{bmatrix}0 & 1\\ 0 & -1\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + \begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}\begin{bmatrix}u_1(t)\\ u_2(t)\end{bmatrix}\\ y(t) &= \begin{bmatrix}5 & 2\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}\begin{bmatrix}0 & 7\end{bmatrix}\begin{bmatrix}u_1(t)\\ u_2(t)\end{bmatrix}\end{aligned}.$$

Use the Sylvester's expansion to determine the state transition matrix and compute the force-free evolution from $\mathbf{x}(0) = (1, 2)^T$.