May 23, 2018 **CK0191: EX 05**

Exercise 01. Consider a discrete-time homogeneous Markov chain with transition probability matrix P

$$P = \begin{array}{c} \\ \mathbf{1} \\ \mathbf{2} \\ \mathbf{3} \\ \mathbf{4} \\ \mathbf{5} \end{array} \begin{array}{c} \begin{array}{ccccc} \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0.8 & 0.2 & 0 & 0 \\ 0.4 & 0 & 0.6 & 0 & 0 \end{pmatrix} \end{array}.$$

i) Plot all possible paths of length 4 that start from initial state **1**; ii) Compute the probabilities of being in each of the states $\{\mathbf{1}, \ldots, \mathbf{5}\}$ four time steps after starting from state **1**.

Exercise 02. Consider a discrete-time homogeneous Markov chain with transition probability matrix P

$$P = \begin{array}{c} \\ \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \\ \mathbf{d} \end{array} \begin{array}{c} \begin{array}{cccc} \mathbf{a} & \mathbf{b} & \mathbf{c} & \mathbf{d} \end{array} \\ \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0.4 & 0.6 & 0 \\ 0.8 & 0 & 0.2 & 0 \\ 0.2 & 0.3 & 0 & 0.5 \end{pmatrix} \end{array}.$$

Compute the following probabilities

i) $\text{Prob}\{X_4 = \mathbf{c}, X_3 = \mathbf{c}, X_2 = \mathbf{c}, X_1 = \mathbf{c} | X_0 = \mathbf{a}\}$;

ii) $\text{Prob}\{X_6 = \mathbf{d}, X_5 = \mathbf{c}, X_4 = \mathbf{b} | X_3 = \mathbf{a}\}$;

iii) $\text{Prob}\{X_5 = \mathbf{c}, X_6 = \mathbf{a}, X_7 = \mathbf{c}, X_8 = \mathbf{c} | X_4 = \mathbf{b}\}, X_3 = \mathbf{d}$.

Exercise 03. Consider a discrete-time Markov chain with transition probability matrix $P(n)$ at times $n = 0, 1, 2, \ldots$

$$P(n) = \begin{array}{c} \\ \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \\ \mathbf{d} \end{array} \begin{array}{c} \begin{array}{cccc} \mathbf{a} & \mathbf{b} & \mathbf{c} & \mathbf{d} \end{array} \\ \begin{pmatrix} 0 & 0.6 & 0.4 & 0 \\ 0.8 & 0 & 0 & 0.2 \\ 0 & 0 & 0.5(0.5)^n & [1 - 0.5(0.5)^n] \\ 0 & 0 & 0.8(0.8)^n & [1 - 0.8(0.8)^n] \end{pmatrix} \end{array}.$$

Compute the probability distribution after two steps if the chain starts from i) state **a**; ii) state **d**.

Exercise 04. Classify the states of the following discrete-time homogeneous Markov chain with transition probability matrix P

$$P = \begin{array}{c} \\ \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \\ \mathbf{d} \\ \mathbf{e} \\ \mathbf{f} \\ \mathbf{g} \\ \mathbf{h} \\ \mathbf{i} \\ \mathbf{j} \end{array} \begin{array}{c} \begin{array}{cccccccccc} \mathbf{a} & \mathbf{b} & \mathbf{c} & \mathbf{d} & \mathbf{e} & \mathbf{f} & \mathbf{g} & \mathbf{h} & \mathbf{i} & \mathbf{j} \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0.3 & 0 & 0.7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.6 & 0 & 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.2 & 0 & 0 & 0 & 0.8 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.9 & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}.$$